



IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #3
VECTORS & MATRICES

Name: Cindy Cheng (10 Peace)

Teacher: Ms. Li, Mr. So & Mr. Wong

Date of task: Friday, December 14, 2012

Time allowed: 95 mins

Student's Performance in Different Criterion			
B	7	D	4

INSTRUCTIONS:

- ◆ Read the instructions for all questions carefully.
- ◆ Show all work, steps and proper units.
- ◆ Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- ◆ Write in **PENCIL**.
- ◆ **NOT** allowed to use any **electronic devices**, such as translators.
- ◆ Allowed to use **GDC**.

ASSESSMENT:

- ◆ Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- ◆ This task assesses Criteria **B & D** considering ALL the questions.

Criterion B: INVESTIGATING PATTERNS (ONLY APPLICABLE TO QUESTION IN PART 1)

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-8)
1-2 Do Maths	The student performs appropriate calculations (in (a) to (c), in (g), to (i), and in (k) to (m)) in order to recognize simple patterns.	The student applies, with some guidance , mathematical problem-solving techniques to recognize simple patterns.	
3-4 General Rule	The student correctly solves (d) and (e) and suggests general rules in parts (f) and (n).	The student <ul style="list-style-type: none"> ● selects and applies mathematical problem-solving techniques to recognize patterns, and ● suggests relationships or general rules. 	Teacher's Final Grade (0-8)
5-6 Test it	The student describes relationships (in (j) and (n)) mathematically, and connects the various relationships. If a student has drawn conclusions consistent with their findings in part 2 (g), credit may be given here.	The student <ul style="list-style-type: none"> ● selects and applies mathematical problem-solving techniques to recognize patterns, ● describes them as relationships or general rules, and ● draws conclusions consistent with findings. 	
7-8 Prove it	The student is able to correctly prove mathematically all the relationships and rules seen in the task, and is successful in (j) and (n).	The student <ul style="list-style-type: none"> ● selects and applies mathematical problem-solving techniques to recognize patterns, ● describes them as relationships or general rules, ● draws conclusions consistent with findings, and ● provides justifications or proofs. 	

Criterion D: REFLECTION (only applicable to question in Part 2).

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-6)
1-2	The student's answers to questions (a) and (b) describe the meaning of his/her findings in real life.	The student attempts to explain whether his or her results make sense in the context of the problem. The student attempts to describe the importance of his or her findings in connection to real life.	
3-4	The student's answers to questions (c) (i) to (iv) comment on how his / her findings make sense in real life situation.	The student correctly but briefly explains whether his or her results make sense in the context of the problem. The student describes the importance of his or her findings in connection to real life where appropriate. The student attempts to justify the degree of accuracy of his or her results where appropriate.	Teacher's Final Grade
5-6	In question (d), the student is able to recognize and explain the applications seen in the vector operations. In question (e), the student has explained how this answer makes sense in the context of this problem.	The student critically explains whether his or her results make sense in the context of the problem. The student provides a detailed explanation of the importance of his or her findings in connection to real life. The student justifies the degree of accuracy of his or her results where appropriate. The student suggests improvements to his or her method where appropriate.	(0-6)

A Special Matrix

Part 1

This section is assessed against criterion B only

You should spend 45-50 minutes on this section, and you are advised to make full use of your GDC.

Consider the following matrices:

$$L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad M = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$

(a) Find the matrix L^2

$$L^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

(b) Find the matrix M^2

$$M^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \\ = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$$

(c) Find the matrix N^2

$$N^2 = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \\ = \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$$

A student, Xavi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ then A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(d) Find A^2

$$\begin{aligned} A^2 &= \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \times \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \\ &= \begin{pmatrix} 2a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & 2a^2+2a+1 \end{pmatrix} \end{aligned}$$

(e) Is Xavi right?

$$\text{If } 2a^2+2a=b$$

$$\begin{aligned} \therefore A^2 &= \begin{pmatrix} 2a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & 2a^2+2a+1 \end{pmatrix} \\ &= \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 3+1 & 3 \\ 3 & 3+1 \end{pmatrix} =$$

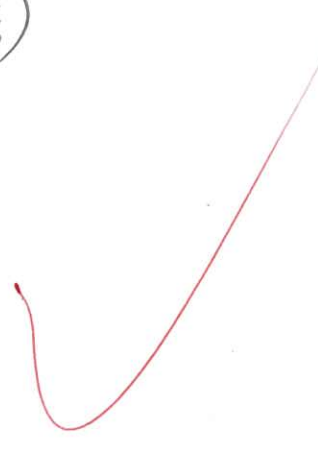
Yes, Xavi is right, A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$.

(f) If your answer to (e) is "yes", is Xavi **always** right? Explain.

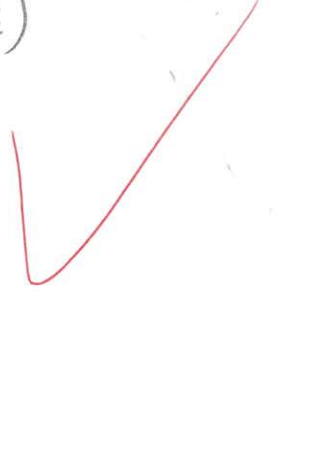
If your answer to (e) is "no", why is he wrong? Explain.

Yes, Xavi is always right, because from questions a, b and c, Xavi's hypothesis is the general formula of L^2 , M^2 and N^2 . Everytime you square a matrix, the result will be $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$, which means justified from the examples from question a, b and c, Xavi's hypothesis is always right.

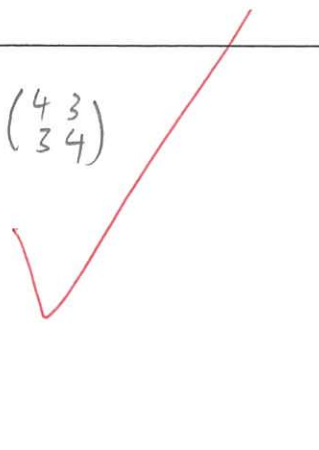
(g) Find the matrix LM

$$\begin{aligned} LM &= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix} \end{aligned}$$


(h) Find the matrix MN

$$\begin{aligned} MN &= \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix} \end{aligned}$$


(i) Find the matrix LN

$$\begin{aligned} LN &= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix} \end{aligned}$$


Another student, Messi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ and B is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

then the matrix AB is always of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(j) Prove (or disprove) Messi's hypothesis

$$AB = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \times \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2ab+a+b+1 & 2ab+a+b \\ 2ab+a+b & 2ab+a+b+1 \end{pmatrix}$$

$$\therefore \text{If } c = 2ab+a+b,$$

$$\therefore AB = \begin{pmatrix} 2ab+a+b+1 & 2ab+a+b \\ 2ab+a+b & 2ab+a+b+1 \end{pmatrix} \\ = \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$$

Now go back to the original matrices.

(k) Find L^3

$$\begin{aligned} L^3 &= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix} \end{aligned}$$

(l) Find L^4

$$\begin{aligned} L^4 &= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix} \end{aligned}$$

(m) Find L^5

$$\begin{aligned} L^5 &= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix} \end{aligned}$$

(n) What do you notice about the form of the answers to (k), (l) and (m)? Try to use the various answers you have in order to generalize.

$$A = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 3(b+1)-1 & 3b+1 \\ 3b+1 & 3(b+1)-1 \end{pmatrix}$$

If $d = 3b+1$

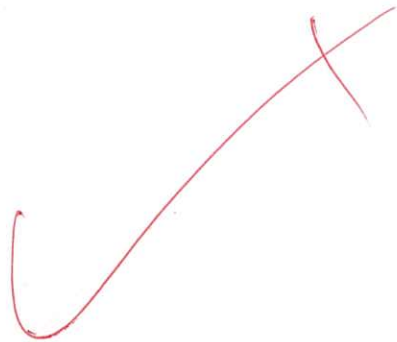
$$A^3 = \begin{pmatrix} d+1 & d \\ d & d+1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} e+1 & e \\ e & e+1 \end{pmatrix}$$

$$A^5 = \begin{pmatrix} f+1 & f \\ f & f+1 \end{pmatrix}$$

For example:

$$L^3 = \begin{pmatrix} (3 \times 4 + 3) - 1 & 3 \times 4 + 1 \\ 3 \times 4 + 1 & (3 \times 4 + 3) - 1 \end{pmatrix} \\ = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$$



Bad guy runs and cop chases... "Catch me if you can!"

Part 2

This section is assessed against criterion D only. You should spend 35-40 minutes on this section.

One day, a thief (T) stole a handbag from a lady (L), the lady shouted for help, a cop (C) was nearby, he then ran after the thief.....

It is given that the position of T, C and L are $(3, 14)$, $(1, 8)$ and $(k, -2k)$ respectively, where k is a constant.

$(-1, 2)$

(a) (i) Express \vec{CT} in terms of **i** and **j**.

$$\begin{aligned}\vec{CT} &= (1+3)\vec{i} + (8+14)\vec{j} \\ &= 4\vec{i} + 22\vec{j}\end{aligned}$$

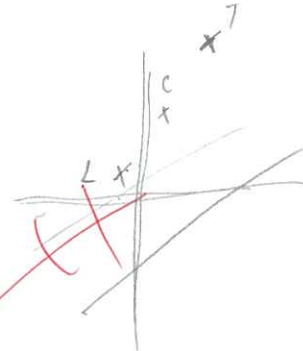
(ii) Express \vec{TL} in terms of **i**, **j** and **k**.

$$\begin{aligned}\vec{TL} &= (k+3)\vec{i} + (-2k+14)\vec{j} \\ &= (k+3)\vec{i} + (14-2k)\vec{j}\end{aligned}$$

- (b) If the value of k is -1 , describe the meaning of this situation in real life. Show your work briefly.

$$\begin{aligned}\vec{TL} &= (-1+3)\vec{i} + (14-2(-1))\vec{j} \\ &= 2\vec{i} + 16\vec{j}\end{aligned}$$

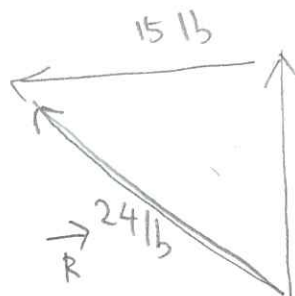
In real life this means that the lady is behind the cop and thief, and she is falling behind.



2(-1)

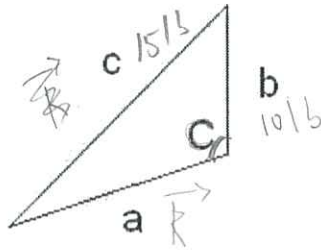
A few minutes later, T and C met... they started fighting for the handbag. Two forces with magnitudes of 15 lb and 10 lb were applied to the bag. The magnitude of the resultant was 24 lb.

- (c) (i) Sketch a diagram to show each force in component form and the resultant force.



- (ii) Find the measurement of the angle between the resultant vector and the vector of the 10 lb force to the nearest degree.

Hint: use cosine rule $c^2 = a^2 + b^2 - 2(a)(b)(\cos C)$



$$\begin{aligned}
 15^2 &= 14^2 + 10^2 - 2(14)(10)(\cos C) \\
 225 &= 676 - 480(\cos C) \\
 225 &= 196(\cos C) \\
 225 - 196 &= \cos C \\
 29 &= \cos C
 \end{aligned}$$

- (iii) According to the above situation, the two forces from different directions are applied to the handbag, can the magnitude of the resultant be larger than the sum of the magnitudes of the forces? Justify your answer.

No.

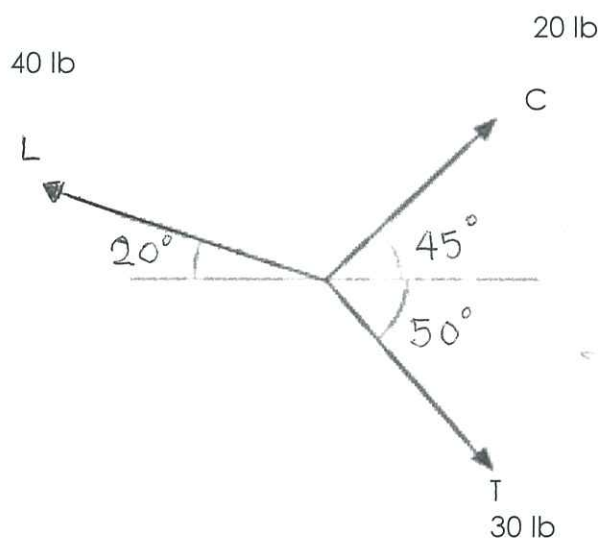
$$15 + 10 = 25$$

\therefore sum is larger than resultant.

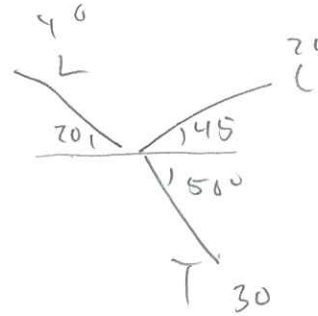
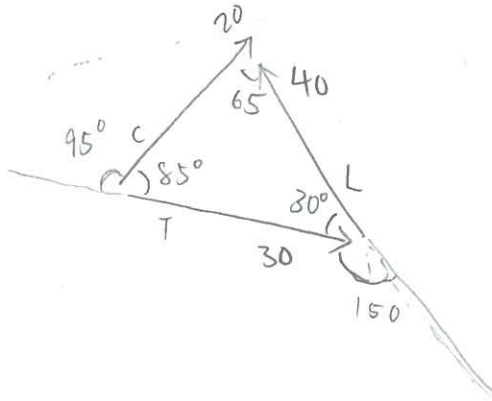
(iv) What if the forces were from the same direction? How would it affect the magnitude of the resultant force? Does it make sense in real life? Briefly justify your answer.

If the forces were from the same direction, it would ^{not} affect the magnitude of the resultant force, because the forces ~~have not change~~ changed. However it does not make sense in real life, because the cop and thief is fighting against each other, and the force would not come from the same direction.

Finally, L caught up with T and C, she joined the fight as well. The diagram shows the 3 forces.



- (d) The three forces shown in the diagram act at a point. Find the magnitude of their resultant and draw a diagram to show its directions. Who would have greater chance to get the handbag?



$$\begin{pmatrix} -40 \cos 115^\circ \\ 40 \sin 115^\circ \end{pmatrix} + \begin{pmatrix} 20 \cos 95^\circ \\ 20 \sin 95^\circ \end{pmatrix} + \begin{pmatrix} 30 \cos 150^\circ \\ -30 \sin 150^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 10.8 \\ 41.2 \end{pmatrix}$$



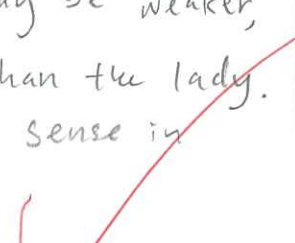
$$41.2^2 + 10.8^2 = \vec{R}^2$$

$$\vec{R} = 42.6$$

Sol.
The Lady has the greater chance to get the handbag, because the resultant force show that the force of lady is.

(e) Explain how this answer makes sense in the context of this problem.

Referring back to the problem, the answer does not seem to make sense, as the lady should be weaker than the cop and thief. The thief may be weaker, however the cop should be stronger than the lady. Therefore the answer does not make sense in context of this problem.



End of Assessment

